UNIT-2

(Lecture-1)

Design of Infinite Impulse Response Digital Filters: Introduction to Filters

Raj Ranjan Prasad

Introduction to Filters

There are several techniques available for the design of digital filters having an infinite duration unit impulse response. The design of an IIR filter involves design of a digital filter in the analog domain and transforming the design into the digital domain.

The system function describing an analog filter may be written as

$$H_a(S) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} \qquad \text{----------(1)}$$

Where a_k and b_k are the filter coefficients.

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The impulse response of these filter coefficients is related to $H_a(s)$ by the Laplace transform.

$$H_a(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt \qquad \text{----------(2)}$$

The analog filter having the rational system function H(s) given in Eq. (1) can also be described by the linear constant-coefficient differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt} \qquad \text{---------(3)}$$

Where x(t) is the input signal and y(t) is the output of the filter. The above three equivalent characterisation of an analog filter leads to three alternative methods for

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transforming the filter into the digital domain. The system function H(s) is stable if all its poles lie in the left-haif of the s-plane.

If the conversion techniques are to be effective then the technique should possess the following properties:-

1.The jω axis in the s-plane should map onto the unit circle in the z-plane. This gives a direct relationship between the two frequency variables in the two domains.

2.The left-half plane of the s-plane should map into the inside of the unit circle in the z-plane to convert a stable analog filter into a stable digital filter.

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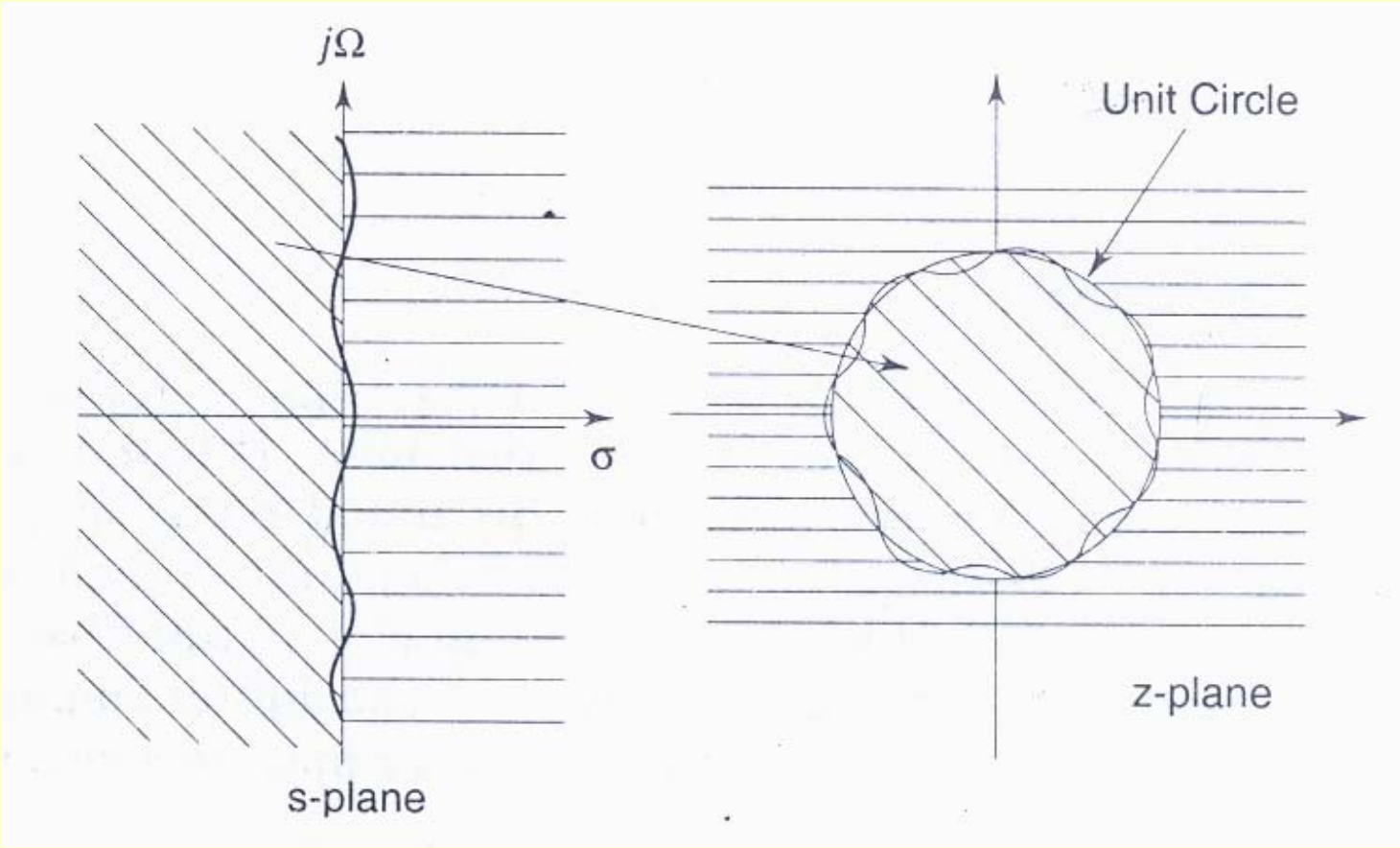

The physically realizable and stable IIR filter cannot have a linear phase.

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For a filter to have a linear phase, the condition is h(n) = h(M-1-n) and the filter would have a mirror image pole outside the unit circle for every pole inside the unit circle. This results in an unstable filter. As\a result, a causal and stable IIR filter cannot have a linear phase. In the design of IIR filters, only the desired magnitude response is specified and the phase response that is obtained from the design methodology is accepted.